Effects of Random Initial Conditions and Deterministic Winds on Simulated Parachute Motion

Guido de Matteis* and Luciano M. de Socio†
Universita' degli Studi "La Sapienza," Rome, Italy

The simulated motion of a parachute subjected to random initial conditions and horizontal wind varying periodically with altitude is considered. The cases of a stable and unstable parachute on a gliding path when one of the initial characteristics of the trajectory is perturbed are discussed. The evolution of the probability density function of one of the Euler angles vs altitude is determined following a small perturbation in this angle. This evolution is evaluated through a stochastic-deterministic approach. Large perturbations are also treated, and the mean value and variance of the Euler angle are calculated. An application ends the paper.

Nomenclature

A = canopy reference area
C_D = drag coefficient
C_N, C_T = canopy normal and tangential force coefficients
$C_{N_\alpha}, C_{T_\alpha}$ = derivatives of force coefficients
g = acceleration due to gravity
i = dimensionless inertia
k = dimensionless mass
m = mass
m_X = mean value
P = probability density function
p,q,r = angular velocity components
R_1 = mass ratio $= (m_c + m_{ch})/m_p$
R_2 = mass ratio $= m_{ch}/(m_c + m_p)$
S = reference surface area
t = dimensionless time
u,v,w = dimensionless velocity components
V = limit velocity of falling sphere
V_c = dimensionless canopy velocity
V_c^* = canopy velocity
V_0^* = velocity of steady glide, reference velocity
V_W = dimensionless wind velocity
X,Y,Z = aerodynamic force components
x,y,z = body axes
x_1,y_1,z_1 = load coordinates
X,Ξ = state vectors
α = angle of attack
θ,ϕ,ψ = Euler angles
ρ = density
σ_X^2 = variance
ω = elementary event in probability space
Ω_t,Ω_z = angular frequencies

Subscripts

c = canopy
ch = canopy hydrodynamic
n = normalized
p = payload
1 = earth axes

I. Introduction

THE purpose of the research reported herein is to evaluate the effects of random initial conditions on the motion of a parachute on a gliding path, including the influence of a horizontal wind, the characteristics of which change with altitude. The problem is of great interest from a practical point of view, since the exact injection of a parachute with its payload on a gliding trajectory and with a prefixed set of initial conditions is virtually impossible. The nature of the problem is essentially probabilistic, especially after one realizes that a completely calm atmosphere or a perfectly steady wind are rather unlikely circumstances.

Depending on its stability characteristics, the motion of a parachute is influenced by the state at the beginning of the glide and by the presence of a variable wind. As a result, the trajectory of the descent and the probability of hitting a given target can be substantially different than when ideal initial conditions are present in a quiescent atmosphere. Further problems can arise when several parachutes are falling, and possible interferences can occur in flight.

In order to carry out this study, some numerical methods will be adopted that have been developed to deal primarily with random motions of systems of particles and with other probabilistic aspects of the mechanics of rigid bodies. As a consequence, a secondary objective of this study is to show how problems of the same mathematical nature as the one presented here can be successfully carried out by means of relatively simple but physically meaningful procedures.

In what follows, the starting point is the deterministic description of parachute motion as developed in Refs. 1 and 2. The mathematical background can be found in a number of textbooks. For example, see Refs. 3–5. More complicated questions, such as those concerned with the description of atmospheric turbulence,[6] will not be discussed.

Sections II and III provide the nondimensional equations of motion and discuss the issue of random initial conditions. With the aim of introducing procedures for solving the equations of motion and determining the probability densities of the dependent variables, simple but meaningful examples are examined.

Next, Sec. IV deals with the case of a parachute subjected to random variations of one of its initial conditions. For the sake of simplicity, variations in the initial conditions of only one of the Euler angles is considered.

Small perturbations are first considered either in the case of a longitudinally stable parachute or in an unstable one. In the case of the stable parachute, a deterministic-stochastic approach is adopted for the solution procedure. With the unstable parachute, the mean and variance of the dependent variables are evaluated as a function of altitude. The section then treats the cases of both stable and unstable parachutes undergoing large random variations in the initial value of the pitch angle.

Section V is devoted to the extension of the analysis to problems arising when a horizontal wind influences the descent of the parachute. The wind will be assumed to vary sinusoidally

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*Research Scientist, Department of Mechanics and Aeronautics. Member AIAA.
†Professor, Department of Mechanics and Aeronautics.

with altitude. Some interesting results are obtained when the frequencies of the variations are close to characteristic frequencies of the parachute-payload system.

In Sec. VI the paper shows an application to a group of parachutes. The probable landing sites are evaluated for randomly distributed launch locations in the presence of wind.

II. Equations of Motion

The basic set of dimensionless equations governing the motion of a symmetric parachute on a gliding path will be stated under the same simplifying assumptions used in Refs. 1 and 2. Those assumptions will not be discussed here. It is essential to keep in mind that, in this model, the number of degrees of freedom of the system formed by the parachute and by the payload is five, since any spinning motion around the axis of symmetry is ignored, i.e., $r=0$.

With reference to Fig. 1, let (x,y,z) and (x_1,y_1,z_1) be two Cartesian axis systems, the first one being the body system with origin 0 at the mass center of the system, and the second one giving the coordinates of the payload in the earth fixed system. All lengths are made nondimensional with respect to the distance between the mass centers of the canopy and of the payload. Velocities are dimensionless with respect to the velocity of steady glide V_0^*.

The pertinent set of equations,[1,2] extended to take into account the effects of horizontal wind, can be written in matrix form as

$$\frac{\mathrm{d}X}{\mathrm{d}t}=F\left(X,t;R_1,R_2,k,i,C_{T_0},\frac{\mathrm{D}V_w}{\mathrm{D}t}\right) \tag{1}$$

where

$$X=(\theta,\psi,x_1,y_1,z_1,u,v,w,p,q) \tag{2}$$

$$F^T=\begin{bmatrix} q\cos\phi \\ q\sin\phi\sec\theta \\ \left(u-\dfrac{qR_1}{1+R_1}\right)\ell_{11}+\left(v+\dfrac{qR_1}{1+R_1}\right)\ell_{21}+w\ell_{31}+V_w \\ \left(u-\dfrac{qR_1}{1+R_1}\right)\ell_{12}+\left(v+\dfrac{qR_1}{1+R_1}\right)\ell_{22}+w\ell_{32} \\ \left(u-\dfrac{qR_1}{1+R_1}\right)\ell_{13}+\left(v+\dfrac{qR_1}{1+R_1}\right)\ell_{23}+w\ell_{33} \\ -\dfrac{C_NV_c^2[u-q/(1+R_1)]}{2k(V_c^2-w^2)^{1/2}}-\dfrac{C_{T_0}}{2k}\sin\theta-qw-\dfrac{\mathrm{D}V_w}{\mathrm{D}t}\ell_{11} \\ -\dfrac{C_NV_c^2[v+p/(1+R_1)]}{2k(V_c^2-w^2)^{1/2}}+\dfrac{C_{T_0}}{2k}\cos\theta\sin\phi+pw-\dfrac{\mathrm{D}V_w}{\mathrm{D}t}\ell_{21} \\ -\dfrac{C_TV_c^2}{2k}-\dfrac{C_{T_0}}{2k}\cos\theta\cos\phi-pw+qu-\dfrac{\mathrm{D}V_w}{\mathrm{D}t}\ell_{31} \\ -\dfrac{C_NV_c^2[v+p/(1+R_1)]}{2i(1+R_1)(V_c^2-w^2)^{1/2}}-\dfrac{C_{T_0}R_2}{2i(1+R_1)}\cos\theta\sin\phi \\ -\dfrac{C_NV_c^2[u-q/(1+R_1)]}{2i(1+R_1)(V_c^2-w^2)^{1/2}}-\dfrac{C_{T_0}R_2}{2i(1+R_1)}\sin\theta \end{bmatrix} \tag{3}$$

where

$$\frac{\mathrm{D}V_w}{\mathrm{D}t}=\frac{\partial V_w}{\partial z_1}\frac{\mathrm{d}z_1}{\mathrm{d}t}+\frac{\partial V_w}{\partial t}$$

$$[\ell_{ij}]=L_{BV}=\begin{bmatrix} \cos\theta\cos\psi & \cos\theta\sin\psi & -\sin\theta \\ \sin\phi\sin\theta\cos\psi-\cos\phi\sin\psi & \sin\phi\sin\theta\sin\psi+\cos\phi\cos\psi & \sin\phi\cos\theta \\ \cos\phi\sin\theta\cos\psi+\sin\phi\sin\psi & \cos\phi\sin\theta\sin\psi-\sin\phi\cos\psi & \cos\phi\cos\theta \end{bmatrix}$$

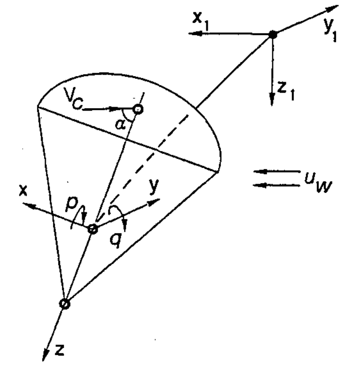


Fig. 1 Sketch of the parachute payload system.

whereas $\phi=\tan^{-1}[\cos\theta\,(d\psi/dt)/(d\theta/dt)]$, since $r=0$.

An initial condition of the form

$$X(t=0)=X_0 \tag{4}$$

is associated with the state equation.

In Eq. (3), the normal and tangential force coefficients C_N and C_T, respectively, are as follows

$$C_N=(X^2+Y^2)^{1/2}/(\tfrac{1}{2}\rho V_c^{*2}A),\qquad C_T=-Z/(\tfrac{1}{2}\rho V_c^{*2}A) \tag{5}$$

Therefore, the dimensionless canopy velocity is given by

$$V_c^2=(V_c^*/V_0^*)^2=w^2+\left(u-\frac{q}{1+R_1}\right)^2+\left(v+\frac{p}{1+R_1}\right)^2 \tag{6}$$

In general, C_N and C_T are experimentally determined functions of the angle of attack of the canopy, i.e., the angle between the symmetry axis and the unperturbed velocity

$$\alpha=\tan^{-1}\left\{\left[\left(u-\frac{q}{1+R_1}\right)^2+\left(v+\frac{p}{1+R_1}\right)^2\right]^{1/2}\Big/w\right\} \tag{7}$$

In this paper one will assume the quadratic forms[1,2]

$$C_N(\alpha)=\alpha_0C_{N\alpha}(\alpha/\alpha_0)(\alpha/\alpha_0-1) \tag{8a}$$

$$C_T(\alpha)=C_{T_0}+\tfrac{1}{2}C_{T_\alpha}\alpha_0(\alpha^2/\alpha_0^2-1) \tag{8b}$$

Finally, in Eq. (3), the characteristic products R_1 and R_2 indicate the relative importance of the masses of the various components of the system, inclusive of the apparent mass of the canopy. The numbers k, i, and C_{T_0} are suitably defined mass, inertia, and weight ratios.

The differential problem of Eqs. (1-4) was extensively studied in the deterministic case and in the case of zero wind. A small disturbance analysis of the longitudinal and lateral stability was conducted, and a conservative glide stability criterion was established. Furthermore, some large disturbance studies and an analysis of the coning were performed. In this work, in addition to accounting for the influence of a variable wind, the initial condition given by Eq. (4) is random. In this context, let $P_0=P(X_0,t=0)$ be the initial probability density function associated with the probability of finding the system in the initial state X_0. The problem is now to determine, if possible, the evolution with t of the probability density function $P(X,t;x_0)$ associated with the probability of finding the system in the state X at the time t starting from the state X_0.

As will be shown later, the evolution of P cannot be determined in the general case, but only when some proper conditions hold. If this is the case, the solution will involve the so-called deterministic-stochastic procedure. Otherwise, one is only able to evaluate the evolution of the mean value and of the variance of X.

Before going into further detail, it is anticipated here that the deterministic-stochastic approach can be adopted only when the deterministic approach predicts stability under small disturbances of the initial state. The description in terms of the mean value and variance of X, which is less detailed, is sought in all other cases. The next section offers two brief significant examples that are fully developed in order to show, in a very simple way, the mathematical procedures used in the more complicated parachute problems.

III. Falling Sphere with Random Initial Conditions

The following examples introduce the questions that are the subject of this research. Let one consider a homogeneous sphere of mass m falling vertically in a calm atmosphere. This situation may represent a model of a parachute in vertical descent. Under well-known assumptions, a limit velocity V can be defined as

$$V=\left(\frac{2mg}{\rho SC_D}\right)^{1/2} \tag{9}$$

For the sake of simplicity both ρ and C_D will be assumed constant. In suitable nondimensional form, if V/g, V^2/g and V are taken as reference time, length, and speed, respectively, the equilibrium equation is

$$\ddot{\xi}=1-\dot{\xi}^2 \tag{10}$$

where ξ is the vertical coordinate, positive downward. Suppose that the initial altitude ξ_0 is known deterministically, whereas the initial velocity $\dot{\xi}_0$ is a random state variable $\dot{\xi}_0(\omega)$.[3] As a consequence, this means that the state

$$\Xi=(\Xi_1,\Xi_2);(\Xi_1=\xi,\quad \Xi_2=\dot{\xi}) \tag{11}$$

is a random process

$$\Xi=\Xi(\omega,t) \tag{12}$$

the evolution of which is governed by the equation

$$\frac{\mathrm{d}\Xi}{\mathrm{d}t}=F(\Xi) \tag{13}$$

where

$$F=(\Xi_2,1-\Xi_2^2) \tag{14}$$

under the initial condition

$$\Xi_0(\omega)=(\Xi_1(t=0)=\xi_0,\ \Xi_2(\omega,t=0)=\dot{\xi}_0] \tag{15}$$

Since F is an analytic function of its arguments, and since the response $\Xi(\omega,t)$ exists and is unique and smooth for every realization of the initial condition $\Xi(\omega)$, then the Jacobian of the map $\Xi(\omega,t)\rightarrow\Xi_0(\omega)$ exists. This means that to any value of ξ at any given t, it is possible to associate a unique value of ξ_0 and vice versa. As a consequence, according to a fundamental theorem of stochastic processes,[3,4] the Jacobian $J=|\partial\Xi_0^T/\partial\Xi|$ is governed by the differential equation

$$\frac{\mathrm{d}J}{\mathrm{d}t}=-J(\nabla\cdot F) \tag{16}$$

where the operation $\nabla\cdot F$ is defined as

$$\nabla\cdot F=\sum_i\frac{\partial F_i}{\partial\Xi_i} \tag{17}$$

with the initial condition $J(t=0)=1$.

Following this information under general hypotheses, the theory of random processes states that the probability density $P[\Xi(t;\Xi_0),\ t]$ associated with the probability of the state Ξ, occurring when the probability density function of the initial state $P(\Xi_0)=P_0$ is assigned, is given by the relation

$$P=P_0J(\Xi\rightarrow\Xi_0,t) \tag{18}$$

In Eq. (18), $J(\Xi\rightarrow\Xi_0,t)$ is read as the value of J at a given t, which corresponds to the initial state Ξ_0.

Now, the system in Eqs. (13) and (16) represents the augmented set of fundamental equations to be solved under the proper initial conditions.

In this first example, since

$$\dot{\xi}=\tanh(t+\tanh^{-1}\dot{\xi}_0)$$

then

$$\frac{\mathrm{d}J}{J}=2\tanh(t+\tanh^{-1}\xi_0)\ \mathrm{d}t$$

$$J=[\cosh^2(t+\tanh^{-1}\dot{\xi}_0)]/[\cosh^2(\tanh^{-1}\dot{\xi}_0)]$$

Therefore, once the initial distribution $\dot{\xi}_0(\omega)$ is given, the probability density function is obtained from Eq. (18). A con-

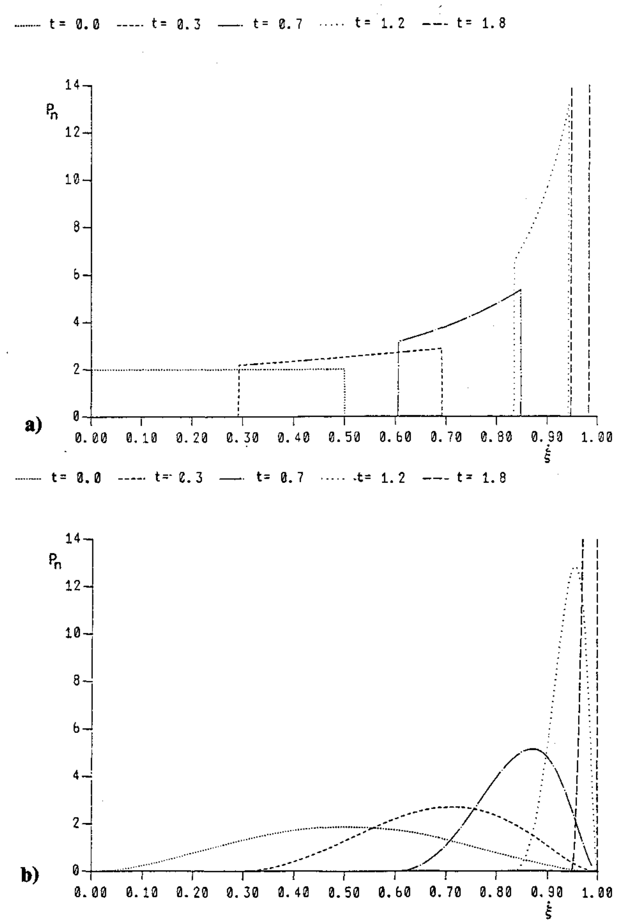


Fig. 2 Evolution of P_n for the falling weight with random initial velocity: a) $P_0=2$ for $0.0\le\dot{\xi}_0\le0.5$; b) $P_0=(5/4)\dot{\xi}_0^2(1-\dot{\xi}_0)^2$, $0.0\le\dot{\xi}_0\le1.0$.

venient representation is the one corresponding to the normalized probability density function

$$P_n(\Xi,t) = P_0[\xi_0(\dot{\xi}_0)]J$$

$$\div\left(\int_{\dot{\xi}(\dot{\xi}_0,\,\min,t)}^{\dot{\xi}(\dot{\xi}_0,\,\max,t)} P_0[\dot{\xi}(\dot{\xi}_0)]\; J[\dot{\xi}(\dot{\xi}_0),t]\; \mathrm{d}\dot{\xi}\right) \tag{19}$$

The actual computations then correspond to consider some initial P_0 and then choose a finite number of initial values of $\dot{\xi}_0$, say $\dot{\xi}_{0,i}$ $(i=1,2,\ldots N)$. Subsequently, $P_n(\dot{\xi}_0,t)$ is evaluated at different times.

Figure 2a shows the evolution of P_n vs $\dot{\xi}$ and t in the case where P_0 is constant for $0 \le \dot{\xi}_0 \le 0.5$, whereas Fig. 2b reports the evolution of P_n in the case where P_0 is given by a β-distribution

$$P_0 = \frac{5}{4}\dot{\xi}_0^2(1-\dot{\xi}_0)^2; \quad \dot{\xi}_{0,\min}=0; \quad \dot{\xi}_{0,\max}=1$$

As one can see in both cases, at increasing times, the speed of maximum probability moves towards unity; in other words, according to the physics of the problem, the sphere shall almost certainly move at a speed very close to the limit speed after a very great time.

In those cases where the state variable X and J cannot be determined analytically, the deterministic-stochastic approach corresponds to numerically solving the augmented system

$$\frac{\mathrm{d}w}{\mathrm{d}t} = G \tag{20}$$

with $w=(X,J)$ $G=(F,-J\nabla\cdot F)$, for a set of initial conditions $w_{0,i}$ $(i=1,\ldots N)$, by means of one of the many initial problem solvers. At the end, the normalized probability density function is computed by numerical quadrature, for the assumed P_0 distribution. The evaluation of J is the stochastic contribution appearing in an otherwise deterministic calculation of X.

As previously stated, the prerequisite for the existence of $J=|\partial X_0^T/\partial X|$ is that the map $X_0 \rightleftharpoons X$, exists at any time. When this requirement is not satisfied, the evolution vs t of the mean value of X and its variance can be followed by calculating

$$m_X = \langle X(\omega,t)\rangle \tag{21a}$$

$$\sigma_X^2 = \langle (X-m_X)^2\rangle \tag{21b}$$

Again, a simple example of this new situation follows.

Let one imagine that a heavy sphere, after falling freely to an altitude z_q, is made to change its limit velocity by a factor of two by changing, for instance, its drag coefficient C_D. Then the equation of motion will be

$$\ddot{\xi} = 1-(\dot{\xi}a)^2$$

with $a=1$, for $t \le t_q = t(z_q)$, and $a=2$ for $t>t_q$. Solutions are reported in Fig. 3a, where it is shown that the relation $\dot{\xi} \to \dot{\xi}_0$, for $t>t_q$ is not a one-to-one correspondence. Therefore, since it is now meaningless to define J, the evolution of the state corresponding to a probability density function for $\dot{\xi}_0$, which is constant for $0 \le \dot{\xi}_0 \le 1$, can be probabilistically followed by observing in Fig. 3b the mean value $m_{\dot{\xi}}$ and the standard deviation $\sigma_{\dot{\xi}}$. For $t\to\infty$, the mean value tends, of course, to the limit speed 0.5 and the standard deviation tends to zero.

IV. Parachute Motion with Random Initial Conditions

It was shown[1,2] that a parachute in gliding motion is longitudinally stable with respect to small perturbations, provided that the aerodynamic derivative $C_{N\alpha}$ satisfies a stability criterion. For the particular set of characteristic numbers assumed throughout this work, the criterion for stability is $C_{N\alpha} > 0.352$.

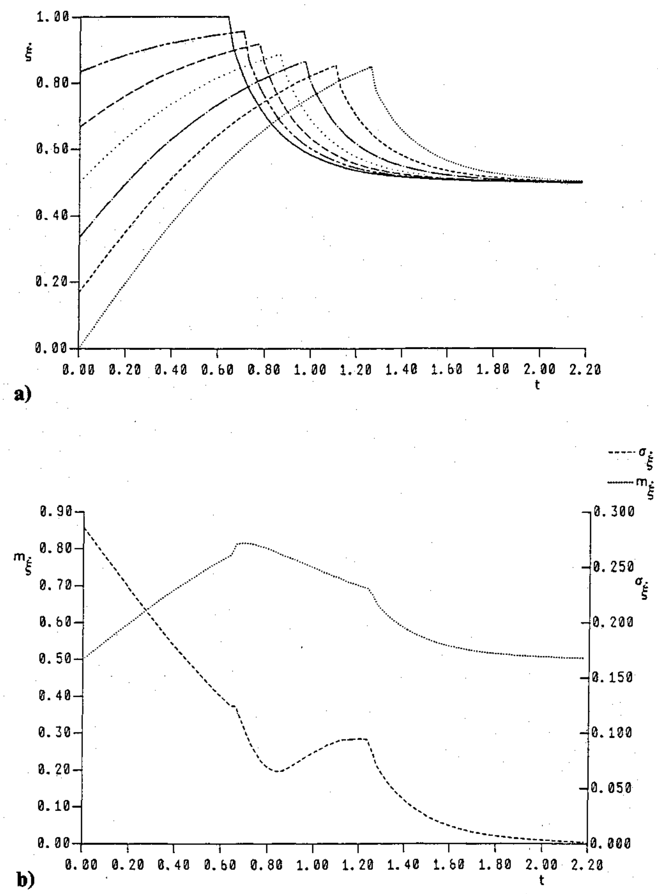


Fig. 3 Falling weight with a variation of a at $z_q = 0.62$: a) Evaluation of $\dot{\xi}(\dot{\xi}_0,t)$; b) mean value and standard deviation of $\dot{\xi}$ vs t.

In this section, the behavior of a longitudinally stable parachute will be considered when small random disturbances of the initial Euler angle θ occur in the absence of wind. Since analytic solutions are not possible, a numerical deterministic-stochastic procedure will be followed. Extensions to perturbations of a parameter other than the pitch angle, which is one of foremost importance in applications, or to more than one parameter, are straightforward in principle.

With this in mind, one considers here the parachute dropped at

$$\begin{vmatrix} x_1(0) \\ y_1(0) \\ z_1(0) \end{vmatrix} = L_{BV}^T \begin{vmatrix} 0 \\ 0 \\ \dfrac{R_1}{1+R_1} \end{vmatrix} \tag{22a}$$

with velocity, respect to the inertial system,

$$\begin{vmatrix} u_1(0) \\ v_1(0) \\ w_1(0) \end{vmatrix} = \begin{vmatrix} \sin\alpha_0 \\ 0 \\ \cos\alpha_0 \end{vmatrix} \tag{22b}$$

and with

$$\phi(0) = p(0) = q(0) = 0; \quad \theta(0) = \theta_0(\omega) \tag{22c}$$

Here, $C_{N\alpha}$ was taken equal to 0.552. Also, the characteristic numbers were assigned the values

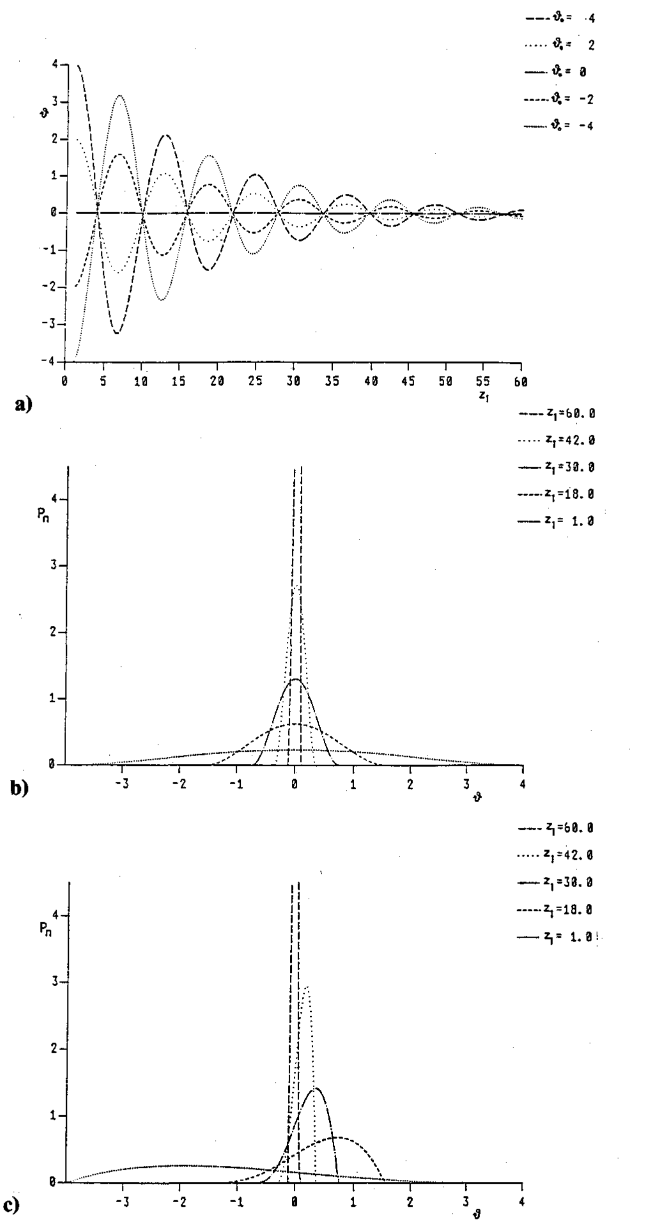


Fig. 4 **Longitudinally stable parachute; small perturbations, C_{N_α} = 0.552: a) $\theta(z_1, \theta_0)$; b) $P_n(z_1, \theta_0)$, symmetric β-distribution; c) $P_n(z_1, \theta_0)$ skewed β-distribution.**

$$R_1 = 0.1;\ R_2 = 0.1;\ i = 1;\ k = 1;\ \alpha_0 = 25 \text{ deg};$$

$$C_{T_0} = 0.7;\ C_{T_\alpha} = -0.1$$

Here and in the following calculations, the random distribution of $\theta_0(\omega)$ corresponds to a β-distribution of $P_0(\theta_0)$, i.e.,

$$P_0 = \frac{a+b+1}{a!b!}\theta_0^a(1-\theta_0)^b$$

and the two situations of a symmetric density function ($a=2$, $b=2$) and of an unsymmetric density function ($a=1$, $b=3$) will be considered.

The steps of the computations were as indicated in the preceding section. A fourth-order Runge-Kutta method was adopted for the integration of the augmented system. The resulting $X(t,\theta_0)$, $J(t,\theta_0)$ and $P_n(t,\theta_0)$ were plotted as $X(z_1,\theta_0)$, $J(z_1,\theta_0)$, $P_n(z_1,\theta)$ due to the one-to-one correspondence of time and altitude. This was done since, from the point of view of practical applications, this was thought to be a convenient representation for an observer concerned with

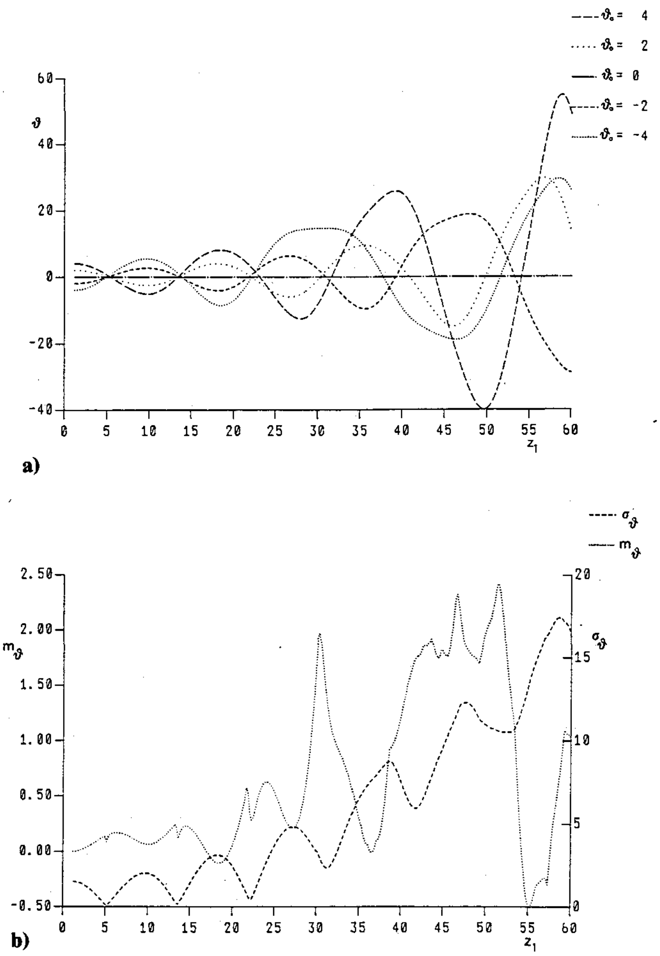


Fig. 5 **Longitudinally unstable parachute; small perturbations, C_{N_α} = 0.152: a) $\theta(z_1, \theta_0)$; b) mean value and standard deviation of $\theta(z_1, \theta_0)$, symmetric β-distribution.**

the characteristics of the parachute descent at fixed altitudes. The distribution of P_n with X and z_1 will appear most useful for evaluating the spread of a group of parachutes on the ground in the final application.

The main results for the stable case are shown in Figs. 4a and 4b, corresponding to a symmetric β-distribution of θ_0, and Fig. 4c for the unsymmetric case.

Figure 4a shows the typical damped oscillations of θ with z_1 for some selected θ_0. Correspondingly, the normalized probability density function moves toward $\theta = 0$ as $z_1 \to \infty$ (Fig. 4b). Similar results can be obtained for the initially skewed probability density function P_0 (Fig. 4c), and again the value $\theta = 0$ is the most probable as $t \to \infty$.

One should note that, since the periodic function $\theta(z_1)$ is multivalued for a set of ordered and countable values of z_1, J cannot be defined. This means that, just for those values, the evaluation of $P_n(z_1)$ is meaningless.

The second situation to be examined was the case of a parachute that is longitudinally unstable to small perturbations in pitch. Now $C_{N\alpha}$ is taken equal to 0.152. The results are presented in Figs. 5a and 5b for a symmetric density function of $P_0(\theta_0)$. Following the observation of the behavior of some sample evolutions of θ vs z_1 (Fig. 5a) with deviations of even greater amplitude, the deterministic-stochastic approach was not practical. The computed trends of m_θ and σ_θ vs z_1 in Fig. 5b show the erratic evolution. It should also be noted that the values of θ tend to become unacceptable very quickly.

The analysis of the large perturbations in the initial pitch angle leads to the results in Figs. 6a anb 6b. Only some results are shown here for a symmetric β-distribution of $P_0(\theta_0)$ in the case $C_{N\alpha} = 0.552$, which corresponds to stability in the small perturbations theory. From the $\theta(z_1,\theta_0)$ evolution in Fig. 6a,

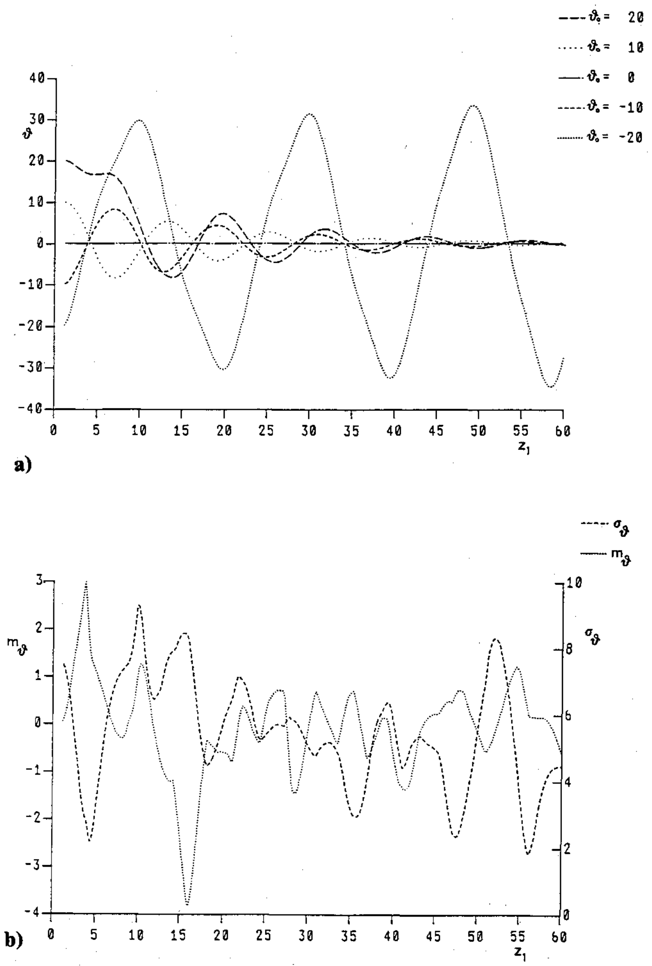


Fig. 6 Longitudinally stable parachute; large perturbations, $C_{N_\alpha} = 0.552$: a) $\theta(z_1, \theta_0)$; b) mean value and standard deviation of $\theta(z_1, \theta_0)$, symmetric β-distribution.

one notes that although the variations of θ damp out more or less rapidly for θ_0 sufficiently small, those corresponding to greater θ_0 do not. As a consequence, as z_1 increases, the mean value and the standard deviation move to zero slowly and erratically. Note the sizeable effects of the large perturbations even though their initial probability is very small due to the β-distribution of θ_0.

V. Wind Effects in the Small Perturbations Analysis

Some interesting results were obtained by including the last term on the left-hand-side of Eq. (3). Here, the results for the case of a wind varying with altitude with the law

$$V_w = [1 + 0.1 \sin(\Omega_z z_1)]$$

will be presented. However, the computations carried out for

$$V_w = [1 + 0.1 \sin(\Omega_t t)]$$

and which are not shown suggest results that are in accordance with what is reported here.

Essentially, the cases of $\Omega_z = 0.1$ or $\Omega_z = 2.0$, far enough from some characteristic resonance frequencies of the system, differ significantly from the cases where these characteristic frequencies are approached. This is shown in Figs. 7a–7c. For the assumed mass and geometry of the parachute-payload couple, the two characteristic frequencies are equal to 0.1942 and 0.4817 respectively.

Figure 7a corresponds to $\Omega_z = 0.1$, and the function $V_w(z_1)$ is represented there for reference. The perturbations on θ are "modulated" by the presence of the wind, and the normalized probabilities displace their maximum values towards a limit oscillating situation.

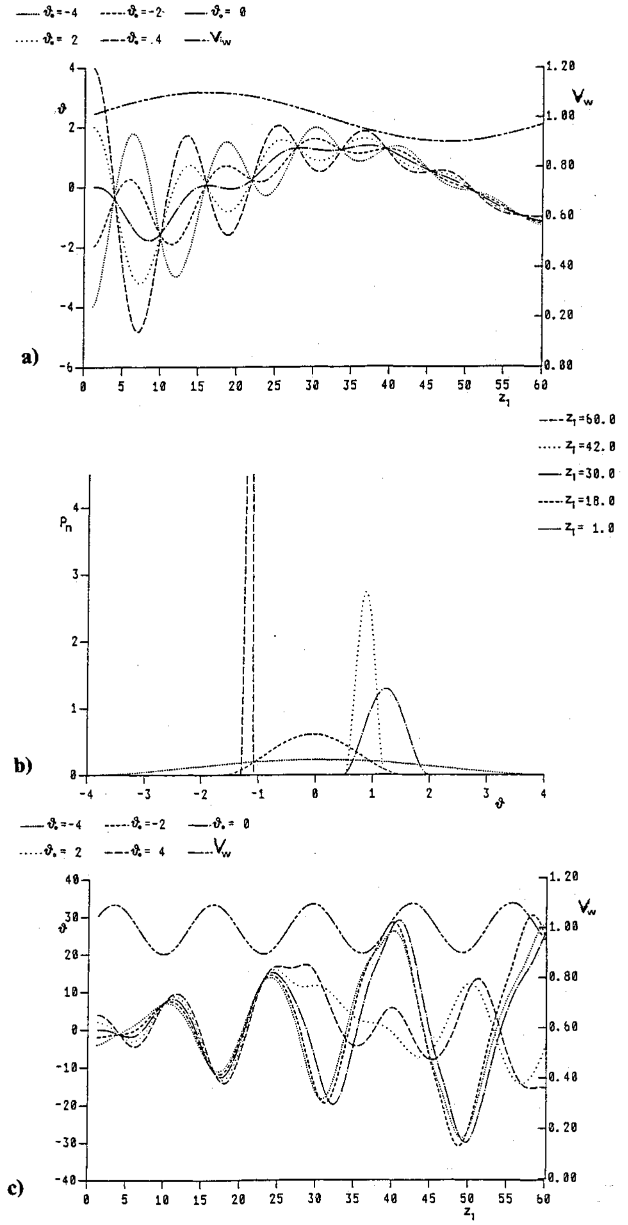


Fig. 7 Longitudinally stable parachute-small perturbations with wind. a) $\theta(z_1, \theta_0)$ at $\Omega_z = 0.1$; b) $P_n(z_1, \theta_0)$; c) $\theta(z_1, \theta_0)$ at $\Omega_z = 0.4817$.

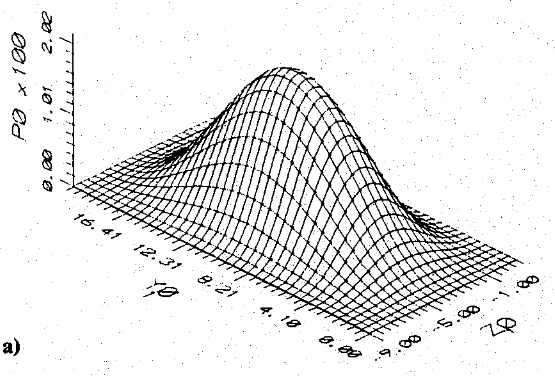
A trend similar to the one shown in Figs. 7a and 7b is also followed by the results calculated for $\Omega_z = 2.0$, which is much higher than the angular frequency in the first case and well outside the resonance range.

Figure 7c corresponds to $\Omega_z = 0.4817$, a resonance value. The parachute is unstable in this condition, as can be easily understood and is confirmed by the calculations of the mean value and of the standard deviation of θ.

When Ω_z equals the second resonance frequency again, instability is observed.

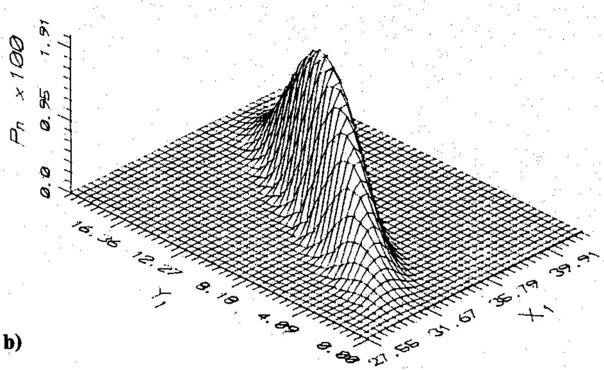
VI. Application

As an application, consider the following problem. A group of identical parachutes is dropped at $t = 0$ in the plane $x_1(0) = 0.5 y_1(0)$ with a random β-distribution of P_0, symmetric about a given mean altitude $\bar{z}_1(0)$. Also, the initial values of $x_1(0)$ and $y_1(0)$ are symmetrically β-distributed around the mean values $\bar{x}_1(0)$ and $\bar{y}_1(0)$. Figure 8a shows P_0 vs y_1 and $z_1(0)$. The same figure depicts P_0 vs $x_1(0)$ and $z_1(0)$ when one



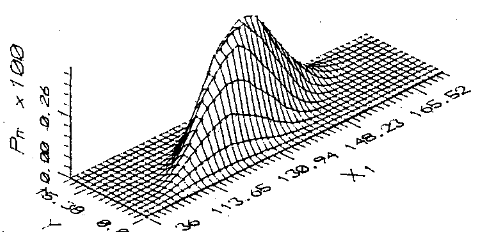
a)

Fig. 8a Probability density of initial locations for a group of parachutes.



b)

Fig. 8b Probability density, on the ground, no wind.



c)

Fig. 8c Probability density, on the ground, in the presence of wind.

reads $x_1(0)$ instead of $0.5\ y_1(0)$. What is the most probable distribution of the landing sites at $z_{1f} = 60$? Consider that the remaining initial conditions are common to the group and expressed by Eqs. (22b) and (22c), but for $\theta_0 = 0$. It is immediately realized that for all the trajectories $y_1(t) = y_1(0)$, both in no-wind and in-wind conditions.

In the no-wind condition with the density P_n, which is a function of x_{1f} and y_{1f}, Fig. 8b will remain unchanged in comparison with $P_0[y_1(0), z_1(0)]$ in the sense that the impact times will change for the single objects, whereas their probable distribution on the ground has the same functional distribution in terms of x_{1f} and y_{1f} that P_0 had in terms of $y_1(0)$ and $z_1(0)$.

When there is a wind that follows the law $V_w(x_1,z_1) = 0.23\ (1+0.03x_1)\ (z_{1f}-z_1)^{0.7}$, the final situation is shown in Fig. 8c. Now, the effect of the wind corresponds to a spreading of the touch points along x_{1f} and to an increase of the probability of landing at shorter x_{1f} than for $V_w = 0$.

Conclusion

In this paper, consideration has been given to the effects of some random perturbations in the initial conditions on the gliding motion of a parachute. Some mathematical tools for dealing with problems of this nature, which frequently arise in flight dynamics, were provided first.

Subsequently the analysis of small and large perturbations of the initial pitch angle was developed in the absence of wind. It was shown how large perturbations of low probability significantly affect the motion of parachutes that meet a longitudinal stability criterion. The analysis was extended to take into account periodic horizontal wind effects on stable parachutes.

As an indication of the capabilities of the presented method, an application concerning the descent of a group of parachutes was reported.

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